Higher order corrections to lensing parameters for extended gravitational lenses

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February 1, 2008

Abstract

We discuss the contribution to the characteristic lensing quantities, i.e. the deflection angle and Einstein radius, due to the higher order terms (e.g. the gravitomagnetic terms) considered in the lens potential. The cases we analyze are the singular isothermal sphere and the disk of spiral galaxies. It is possible to see that the perturbative effects could be of the order 10^{-3} with respect to the ordinary terms of weak field and thin lens approximations, so that it is not a far hypothesis to obtain evidences of them in a next future by suitable experiments.

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1   Introduction

In the last decades, gravitational lensing has revealed a powerful tool in astrophysics and cosmology so that a newborn branch of astronomy has been named "gravitational astronomy". Some basic assumptions are always taken into account in the theory: the first is the weak field approximation. It is due to the fact that the most of gravitational systems considered, as lenses and sources, are well beyond their Schwarzschild radii so that strong regimes can be ignored. Besides, thin lens approximation is often used since the thickness of deflecting systems is very small with respect to the distances between observer, lens, and source respectively. A further simplification is related to the fact that deflection angles are, in general, small. The result of these approximations is that the theory of gravitational lensing is quite simple [1] and the main interests are devoted to the development of applications.

However, gravitational lensing must not be conceived as a weak field phenomenon since the bending and the looping of light rays are predictions of "full" General Relativity theory. Besides this fact of principle, several facts and observations ask for the enlargement to any order of approximation. For example, when the lenses are compact objects, as black holes or pulsars, the above approximations never hold so that more general and involved schemes have to be adopted. In these cases, the gravitational systems are close to their Schwarzschild radii and the weak field approximation fails [2],[3]. A full treatment of lensing phenomena gives rise to a transcendent lens equation which has to be handled with some care in order to be solved [3].

Further considerations deserve the weak field limit which could give interesting results if it is not simply considered at lowest orders. In [4], it was shown that taking into account corrections of order v/c gives rise to nonnegligible effects which becomes more and more interesting in a cosmological context. Manzano and Montemayor [5] develops the full theory of the propagation of light in a gravitational background showing that several effects, as polarization, depends on the intrinsic feature of the gravitational field of the lens. It is worthwhile to note the so called Rytov effect [6], i.e. the rotation of the plane of polarization of the incident beam which is generated by the gravitational field of a rotating mass. In this effect, the angular momentum gives contribution to the deflection angle. In general, as it is widely discussed in [4], taking into account the gravitomagmetic term introduces corrections which are useful if we consider the proper motions of the lens. Their magnitude can be relevant depending on the ratio v/c.

As a further consideration, the study of higher order perturbative terms is the link between weak and strong regimes which can allow the full parametrization of lensing theory.

Finally, a close comparison between General Relativity and other relativistic theories of gravity can be led on the base of higher order effects since the differences between them could not emerge in the lowest order of the weak field limit [7],[8],[9].

In this paper, we take into account the gravitomagnetic contributions to the lensing quantities, as the deflection angle and the Einstein radius, for extended gravitational

lenses of astrophysical interest as the singular isothermal sphere and the disk of spiral galaxies. In both cases, the proper motions (i.e. the velocity dispersion and the circular velocity) have to be taken into account and, in some peculiar cases (e.g. AGN, or other kinds of extreme active galaxies) their contributions could be far to be trivially ignored.

However, the singular isothermal sphere as lens was considered by many authors. For example, Gurevich and Zybin [10] use it to study structures of cold dark matter at small scale regions. They give a constraint for the minimal dark matter objects (which should be in the range $M_X \sim 10^{-4} \div 10^{-2} M_\odot$) and another for the particle candidate (neutralino) $m_X \sim 10 \pm 7$ GeV. Sazhin et al. [11] adopt singular isothermal sphere models to study microlensing by non-compact invisible objects. They discuss the OGLE collaboration results towards Large Magellanic Cloud which point out the presence of dark bodies with a mass of the order of $0.1 M_\odot$. The authors suggest, besides the standard hypothesis that such objects are brown dwarfs, other hypotheses which includes mirror stars, black holes, and exotic stars which consist of cold dark matter particles. In particular, they discuss the last possibility which is attractive from the point of view of modern particle physics and find that some observational data can be consistent with it.

In Sect.2, we discuss the case of singular isothermal sphere and in Sect.3, the disk of spiral galaxies is taken into account. The results are discussed in Sect.4.

2   The singular isothermal sphere

Isothermal sphere is the simplest model used to describe the mass function of the haloes of galaxies and to derive the potential of elliptical galaxies [12]. As a lens model, it is the further step after the pointlike Schwarzschild lens. The internal motions (e.g. the velocity dispersions, proper motions of the stars, etc.) give rise to nontrivial effects capable of supporting the dynamics of the real systems [12]. If the system described by an isothermal sphere acts as a lens, these effects can lead to gravitomagnetic corrections which could be quantitatively significant.

The mass density for a singular isothermal sphere is given by

$$\rho = \frac{\sigma_v^2}{2\pi G x^2} \tag{1}$$

with

$$\vec{x} = \vec{\xi} + l\vec{e}_{in} \tag{2}$$

where σ_v is the velocity dispersion of the lens, ξ is the distance from the centre of the sphere, $\vec{e}_{in}$ is a unitary vector in the initial direction of light. Gravitational potential is given by

$$\phi(x) = -G \int d^3 x' \frac{\rho(x)}{|\vec{x} - \vec{x'}|} = -\frac{\sigma_v^2}{2\pi} \int d^3 x' \frac{1}{x'^2} \frac{1}{|\vec{x} - \vec{x'}|};, \tag{3}$$

which is

$$\phi(x) = -2\sigma_v^2 \ln\left(\frac{x}{R}\right) = -2\sigma_v^2 \ln\left(\frac{\xi + le_{in}}{R}\right) \tag{4}$$

where R is a cut-off distance introduced to eliminate the singularity in the origin.

A vector potential can be defined as

$$\vec{V} = \vec{v}\phi \tag{5}$$

where $\vec{v}$ is a velocity. The k-component results

$$V_k = -2\sigma_v^2 v_k \ln\left(\frac{\xi + le_{in}}{R}\right). \tag{6}$$

The deflection angle of light, taking into account also the potential vector term [1],[4] is given by

$$\vec{\alpha} = \frac{2}{c^2} \int \vec{\nabla}_\perp \phi \, dl - \frac{4}{c^3} \int (\vec{e} \wedge (\vec{\nabla} \wedge \vec{V})) dl, \tag{7}$$

where dl is the Euclidean line element. The last term in the right-hand side is the gravitomagnetic correction. If we solve for every single vector component, evaluating the integrals between 0 and ∞, we get

$$\alpha_k = -\frac{4\sigma_v^2 \xi_k}{c^2} \int_0^\infty \frac{1}{\xi^2 + l^2} dl - \frac{8\sigma_v^2 v_k}{c^3} \int_0^\infty \frac{l}{\xi^2 + l^2} dl \tag{8}$$

which results

$$\alpha_k = -\frac{2\sigma_v^2 \pi}{c^2} \frac{\xi_k}{\xi} - \frac{4\sigma_v^2 v_k}{c^3} \ln\left[1 + \left(\frac{R}{\xi}\right)^2\right]. \tag{9}$$

The last term in (9) is due to the gravitomagnetic correction and it clearly depends by the ratios v_k/c (i.e. the kinematics) and R/ξ (i.e. the geometry). It is straightforward to see that the correction is significant only for appreciable values of these ratios. For example, it is easy to estimate that for $v_k/c \simeq 10^{-(2\div3)}$ and $R \sim \sqrt{2}\xi$ we could appreciate some effects. This means that high proper motions and the impact parameters of the light beams comparable to the physical sizes of the lenses can give rise to appreciable gravitomagnetic corrections.

3 The disk of spiral galaxies

Gravitomagnetic corrections to the disk potential of a spiral galaxy are another interesting case. The k-component of the deflection angle is

$$\alpha_k^{grav}(\xi) = -\frac{4}{c^3} \int [\vec{e} \wedge (\vec{\nabla} \wedge \vec{V})]_k dl = -\frac{4}{c^3} \int \left[\partial_k(\vec{e} \cdot \vec{V}) - (\vec{e} \cdot \vec{\nabla})_k \cdot \vec{V}\right] dl. \tag{10}$$

Solving the integral in polar coordinates

$$dl \equiv (dx, dy) \rightarrow (d\xi, \xi d\theta) \tag{11}$$

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and taking in account that the potential does not depend on θ for symmetry properties, the integration becomes simply depending by ξ. It is easy to find that

$$\alpha_k^{grav}(\xi) = \frac{4}{c^3} v_k \int \frac{d\psi(\xi)}{d\xi} d\xi \tag{12}$$

in which $\psi(\xi)$ is the gravitational potential. The mass distribution of a spiral galaxy is

$$M(\xi) = 2\pi \xi_c^2 \Sigma_0 \left[1 - \left(1 + \frac{\xi}{\xi_c} \right) e^{-\frac{\xi}{\xi_c}} \right] \tag{13}$$

where ξ_c is a scale length normally projected along the line of sight [1].

The deflection angle for a disk-like deflector is:

$$\alpha(\xi) = \frac{4GM(\xi)}{c^2 \xi} = \frac{8\pi G \xi_c^2 \Sigma_0}{c^2} \left[\frac{1}{\xi} - \frac{\left(1 + \frac{\xi}{\xi_c} \right) e^{-\frac{\xi}{\xi_c}}}{\xi} \right] . \tag{14}$$

The potential is given by

$$\psi(\xi) = - \left(\frac{D_{ds}}{D_d D_s} \right) \int \alpha(\xi) d\xi + \psi_0 , \tag{15}$$

and then

$$\psi(\xi) = \psi_0 - \left(\frac{D_{ds}}{D_d D_s} \right) \frac{8\pi G \xi_c^2 \Sigma_0}{c^2} \left\{ e^{-\frac{\xi}{\xi_c}} + \frac{\xi}{\xi_c} - \frac{1}{4} \left(\frac{\xi}{\xi_c} \right)^2 + \frac{1}{18} \left(\frac{\xi}{\xi_c} \right)^3 + ... \right\} \tag{16}$$

where D_{ds}, D_d, D_s are the deflector-source, the deflector-observer, and the source-observer distances respectively.

Immediately, we see that the correction to α due to the gravitomagnetic term is given by

$$\alpha_k^{grav}(\xi) = \frac{4}{c^3} v_k \psi_0 - \left(\frac{D_{ds}}{D_d D_s} \right) \frac{32 v_k \pi G \xi_c^2 \Sigma_0}{c^5} \left\{ e^{-\frac{\xi}{\xi_c}} + \frac{\xi}{\xi_c} - \frac{1}{4} \left(\frac{\xi}{\xi_c} \right)^2 + \frac{1}{18} \left(\frac{\xi}{\xi_c} \right)^3 + ... \right\} \tag{17}$$

which is, in a compact form,

$$\alpha_k^{grav}(\xi) = \frac{4}{c^3} v_k \psi_0 - \left(\frac{D_{ds}}{D_d D_s} \right) \frac{32 v_k \pi G \xi_c^2 \Sigma_0}{c^5} \left[e^{-\frac{\xi}{\xi_c}} - \sum_{n=1}^{\infty} (-1)^n \frac{1}{nn!} \left(\frac{\xi}{\xi_c} \right)^n \right] . \tag{18}$$

Also in this case, the role of the ratios $\frac{v_k}{c}$ and $\frac{\xi}{\xi_c}$ is leading to appreciate the correction.

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4   Discussion and Conclusions

In this paper, we have analyzed, in the framework of gravitational lensing theory, the effects of gravitomagnetic corrections on the deflection angle in the case of extended mass lenses. In [4], gravitomagnetic corrections have been taken into account for point-like deflectors and, in that case, the deflection angle is corrected by a factor which can be related to the redshift of the lens. It is clear that the possibility to detect the correction depends on the fact that point-like lenses should have a sufficiently high redshift (which could be intrinsic in the case of high proper motions).

These results can be generalized to extended lens models where the effects of proper motions can be very relevant and then the vector potential terms in the perturbative expansion of gravitational field are not negligible.

In particular, we have analyzed two mass models: the singular isothermal sphere and the disk of spiral galaxies.

In the first case, we have found that the gravitomagnetic correction depends on the ratios v_k/c and R/ξ, that is on the kinematics and the geometry of the system. Then it is still possible to link this term to the redshift so that we can give a quantitative evaluations of gravitomagnetic corrections. To give relevant effects, the second term in Eq.(9) should be of the order $10^{-(2\div3)}$ with respect to the first. These are the limits set by the forthcoming space and ground-based experiments [13]. It is easy to see that these constraints can be achieved by taking into account exotic objects (e.g. AGN) as lenses. It is worthwhile to stress the fact that such constraints could be used also to study exotic non-compact invisible bodies [10],[11] by taking into account their kinematics.

In the case of the disk of spiral galaxies, we find that the correction to the deflection angle depends on exponential term (decreasing according to a scale factor) and on an infinite series of terms depending on the same scale factor [see Eq.(17)]. Similarly to the above situation, geometry and kinematics lead the corrections and can be appreciated at certain scales. This fact is in agreement with the results of several relativistic theories of gravity, where the corrections to General Relativity are relevant depending on the interaction scales (see e.g. [8],[9],[14],[15]). In those cases, the Newtonian potential has to be corrected by Yukawa terms or power law series, strictly dependent on some characteristic scale (e.g. given by the mass of some interaction boson). For example, considering the conformal gravity, Mannheim et al. [14] "derive" the flat rotation curve of spiral galaxies without using huge amounts of dark matter. However, also lensing quantities are affected in these extended theory of gravity (see [8],[9]). For example, in [16] a systematic analysis of light deflection and time delay is performed in the context of Weyl gravity. The main result is that all lensing quantities are corrected and several observations, which cannot be fitted in General Relativity context, could be reinterpreted in this scheme. From our point of view, higher order corrections in the series expansion of gravitational field acquire a similar role since they reproduce, at least from a formal point of view the same results. In this sense, they have to be considered if relativistic effects are not negligible.

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